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Production Inventory Model for Perishable Items with Exponential Declining Demand and Partial Backlogging

Abstract

Perishable products constitute a sizable component of inventories. When a product is highly perishable, the demand may need to be backlogged to contain costs due to deterioration. In this paper problem of determining the optimal policy for perishable items with life time has been considered. The demand for the product is taken as exponential decreasing function of time and the production rate is the linear combination of on-hand inventory and demand rate. Shortages are allowed and partially backlogged and the backlogging rate is taken as dependent on the duration of waiting time. Finally, numerical examples are also used to study the behaviour of the model.

KEYWORDS: Exponential Declining Demand, Deterioration, Life Time, Partial Backlogging.

1. INTRODUCTION:

The problem of deteriorating inventory has received considerable attention in recent years. Deterioration is defined as change, damage, decay, spoilage obsolescence and loss of utility or loss of original value in a commodity that results in the decreasing usefulness from the original one. It is well known that certain products such as vegetable, medicine, gasoline, blood and radioactive chemicals decrease under deterioration during their normal storage period. As a result, while determining the optimal inventory policy of that type of products, the loss due to deterioration cannot be ignored. In the literature of inventory theory, the deteriorating inventory models have been continually modified so as to accumulate more practical features of the real inventory systems. Most researchers in deteriorating inventory have assumed constant rate of deterioration. However the variable rate is used to represent the deterioration for the products which deteriorates with time in stock. The deterioration rate increase with age, that is, the longer the items remain unused higher the rate at which they fail. **Wee (1999)** discussed a deterministic inventory model with quantity discount, pricing and partial backlogging when the product in stock deteriorates with time. **Papachristos** and **skouri (2003)** generalized the work of **Wee (1999)** and considered a model where the demand rate is a convex-decreasing function of the selling price and the backlogging rate is a time dependent function. **Abad (2003)** considered the pricing and lot-sizing problem for perishable items with finite production, exponential decay and partial backordering. **Goyal** and **Giri (2003)** developed the production inventory model for perishable goods with time varying demand and production. In most of the inventory models, it is assumed that the deterioration occurs as soon as the retailer receives the commodity but for many items this is not true. In real life, most of the items arrive in stock they are fresh and new and they begin to decay after a fixed time interval called life-period of items. **Wu et al. (2006)** discussed an optimal replenishment policy for such non-instantaneous deteriorating products with stock dependent demand and partial backlogging.

However, in most of the inventory models it is unrealistically assumed that during stockout, either all demand is backlogged or all is lost. In reality often some customers are willing to wait until replenishment, especially if the wait will be short, while others are more impatient and go elsewhere. The backlogging rate depends on the time to replenishment-the longer customers must wait, the greater the fraction of lost sales. **Abad (1996)** developed a pricing and lot-sizing EOQ model for a product with a variable rate of deterioration and partial backlogging. **Abad (2000)** then extended the optimal pricing and lot-sizing EOQ model to an economic production quantity (i.e. EPQ) model. **Papachristos** and **Skouri (2000)** discussed an optimal replenishment policy for deteriorating items with time-varying demand and partial-exponential-type backlogging. Other articles related to this research were written by **Ouyang et al. (2005)**, **Singh** and **Singh (2007)**, **Yang (2007)**, **Singh et al. (2008)** and so on.

In the present paper an attempt has been made to study the situations in which demand decreases exponentially. It is commonly observed that the demand of an item declines over time because of the continuous introduction of competing products, loss of appeal, change in trend or perception about the product and so on for instance, fashion goods grow out of vogue after some time. The demand for a new product like a new model of a computer can not be continuously sustained when newer and more efficient products are introduced into market. The demand for seasonal products like winter clothes decrease over the season. Therefore, the concept of exponential decreasing demand is a very realistic in real life situations. **Kumar** and **Sharma (2000)** developed inventory model for decaying items with such exponential demand.

This paper is concerned with the development of production inventory model for deteriorating items with the concept of life time. In this model the demand for the product is taken as exponential decreasing function of time and the production rate is the linear combination of on-hand inventory and demand rate at any time. Shortages are allowed and partially backlogged and the backlogging rate is taken as variable and dependent on the duration of waiting time for the next replenishment and varies inversely. Formulations of inventory policies with such combination have seldom been mentioned. Finally, numerical examples are also used to study the behaviour of the model and the effects due to change in various parameters have been considered in the model numerically.

2. ASSUMPTIONS AND NOTATIONS:

In developing the mathematical model of the production inventory system for this chapter, the following assumptions and notations are being made:

(i) I (t) is the inventory level at any time t, $t \geq 0$.

(ii) C', C_1, C_2, C_3, C_4 denote the set up cost for each replenishment, inventory carrying cost per unit time, deterioration cost per unit, shortage cost for backlogged items and the unit cost of lost sales respectively. All of the cost parameters are positive constants.

(iii) Lead time is zero and no replenishment or repair of deteriorated items is made during a given cycle.

(iv) A single item is considered over the prescribed period T units of time, which is subject to variable deterioration rate.

(v) Deterioration of the items is considered only after the life time of items.

(vi) μ is the life time of items and $\theta(t)$ is the variable deterioration rate s.t. $\theta(t) = \alpha t,\ 0 < \alpha << 1$.

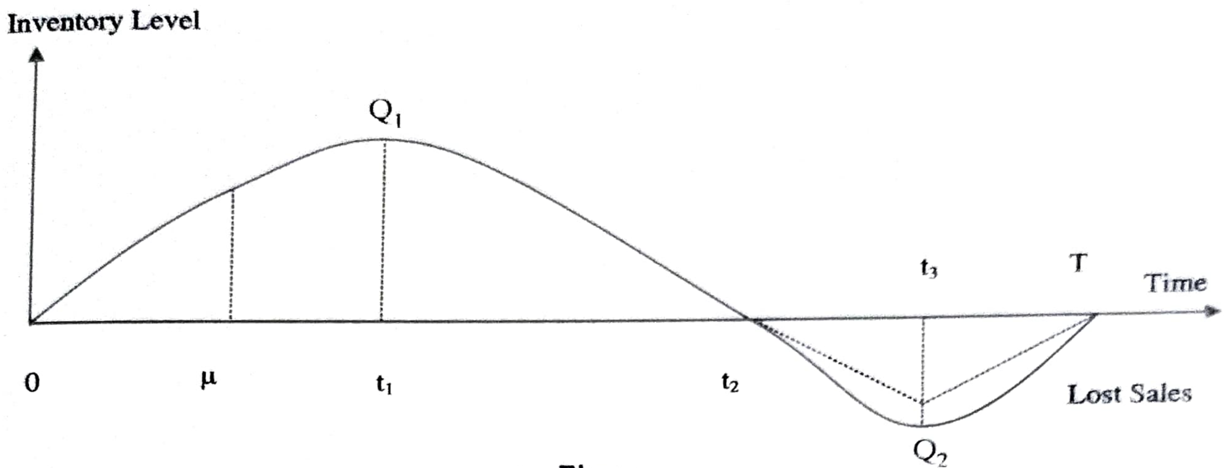
(vii) Demand rate D(t) is known and decreases exponentially s.t. $D(t) = \beta e^{-\lambda t}, t \geq 0$. β is the initial demand, λ is a constant and responsible for decreasing rate of demand.

(viii) Production rate is the linear combination of on-hand inventory and demand rate at any time s.t. $P(t) = I(t) + bD(t)$, $b > 1$ and $P(t) > D(t)$.

(ix) Shortages are allowed and the backlogging rate is $\frac{\beta e^{-\lambda t}}{1+\gamma(T-t)}$, when inventory is in shortage. The backlogging parameter γ is a positive constant s.t. $0 < \gamma << 1$.

3. FORMULATION AND SOLUTION OF THE MODEL:

Initially, the inventory level is zero. The production starts at time t = 0 with the concept of life time μ, when the inventory level become q deterioration can take place and after t_1 units of time, it reaches to maximum inventory level Q_1. After this, production stopped and at time t_2, the inventory becomes zero. At this time shortage starts developing and at time t_3 it reaches to maximum shortage level Q_2, at this time fresh production starts to clear the backlog by the time t_4. Here our aim is to find the optimum values of t_1, t_2, t_3 and t_4 that minimize the total average cost (K) over the time horizon (0, T). The depletion of inventory is given in the figure.



Figure

The inventory level I(t) at time $t (0 \le t \le T)$ satisfies the differential equations:

$$I'(t) = P(t) - D(t), \qquad 0 \le t \le \mu \qquad \ldots (1)$$

$$I'(t) = P(t) - \theta(t) I(t) - D(t), \qquad \mu \le t \le t_1 \qquad \ldots (2)$$

$$I'(t) = -\theta(t) I(t) - D(t), \qquad 0 \le t \le t_2 \qquad \ldots (3)$$

$$I'(t) = -\frac{D(t)}{1 + \gamma(T - t)}, \qquad 0 \le t \le t_3 \qquad \ldots (4)$$

and $$I'(t) = P(t) - D(t), \qquad 0 \le t \le t_4 \qquad \ldots (5)$$

The boundary conditions are

$$I(t) = 0 \text{ at } t = 0,\ t_1 + t_2 \text{ and } T (= t_1 + t_2 + t_3 + t_4) \qquad \ldots (6)$$

$$I(\mu) = q,\ I(t_1) = Q_1 \text{ and } I(t_1 + t_2 + t_3) = Q_2 \qquad \ldots (7)$$

Since $P(t) = I(t) + bD(t)$ and $D(t) = \beta e^{-\lambda t}$, therefore the solutions of above equations are given by

$$I(t) = \frac{(b-1)\beta}{(\lambda+1)} (e^{t} - e^{-\lambda t}), \qquad 0 \le t \le \mu \qquad \ldots (8)$$

$$I(t) = (1-b)\beta \frac{e^{-\lambda t}}{(\lambda+1)} \left[1 + \alpha \left\{ \frac{t}{(\lambda+1)} + \frac{1}{(\lambda+1)^2} \right\} \right] + q e^{t-\mu} \left\{ 1 + \frac{\alpha}{2} (\mu^2 - t^2) \right\}$$

$$- (1-b)\beta \frac{e^{t-(\lambda+1)\mu}}{(\lambda+1)} \left[1 + \alpha \left\{ \frac{1}{2} (\mu^2 - t^2) + \frac{\mu}{(\lambda+1)} + \frac{1}{(\lambda+1)^2} \right\} \right], \mu \le t \le t_1 \qquad \ldots (9)$$

$$I(t) = \frac{\beta}{\lambda} \left[e^{-\lambda t} \left\{ 1 + \alpha \left(\frac{t}{\lambda} + \frac{1}{\lambda^2} \right) \right\} - e^{-\lambda t_2} \left\{ 1 + \alpha \left(\frac{1}{2} (t_2^2 - t^2) + \frac{t_2}{\lambda} + \frac{1}{\lambda^2} \right) \right\} \right], \quad 0 \le t \le t_2 \qquad \ldots (10)$$

$$I(t)=\frac{\beta}{\lambda}\left[\left\{1+\gamma\left(t-T+\frac{1}{\lambda}\right)\right\}e^{-\lambda t}-\left\{1+\gamma\left(\frac{1}{\lambda}-T\right)\right\}\right],\ 0\le t\le t_3 \quad \dots (11)$$

and $$I(t)=\frac{(b-1)\beta}{\lambda+1}\left\{e^{t-(\lambda+1)t_4}-e^{-\lambda t}\right\},\qquad 0\le t\le t_4 \quad \dots (12)$$

respectively.

Now by using boundary condition (7) and then by equation (9), (10) and (11), (12), we get

$$Q_1=\frac{\beta}{\lambda}\left(1+\frac{\alpha}{\lambda^2}\right)-\frac{\beta}{\lambda}e^{-\lambda t_2}\left\{1+\alpha\left(\frac{t_2^2}{2}+\frac{t_2}{\lambda}+\frac{1}{\lambda^2}\right)\right\}=(1-b)\beta\frac{e^{-\lambda t_1}}{\lambda+1}\left[1+\alpha\left\{\frac{t_1}{\lambda+1}+\frac{1}{(\lambda+1)^2}\right\}\right]$$

$$+qe^{t_1-\mu}\left\{1+\frac{\alpha}{2}(\mu^2-t_1^2)\right\}-(1-b)\beta\frac{e^{t_1-(\lambda+1)\mu}}{\lambda+1}\left[1+\alpha\left\{\frac{1}{2}(\mu^2-t_1^2)+\frac{\mu}{\lambda+1}+\frac{1}{(\lambda+1)^2}\right\}\right] \quad \dots (13)$$

and $$Q_2=\frac{\beta}{\lambda}\left[\left\{1+\gamma\left(t_3-T+\frac{1}{\lambda}\right)\right\}e^{-\lambda t_3}-\left\{1+\gamma\left(\frac{1}{\lambda}-T\right)\right\}\right]=\frac{(b-1)\beta}{\lambda+1}\left\{e^{-(\lambda+1)t_4}-1\right\} \quad \dots (14)$$

From these equations, we observe that variables t_1, t_2 and t_3, t_4 are dependent variables. Therefore we can write

$$t_2=f_1(t_1)\text{ and }t_3=f_2(t_4) \quad \dots (15)$$

Total amount of deteriorated units (I_D) during the period (0, T) is given by

$$I_D=\int_{\mu}^{t_1}\theta(t)I(t)dt+\int_{0}^{t_2}\theta(t)I(t)dt$$

$$=\frac{(b-1)\alpha\beta}{\lambda+1}\left[\frac{1}{\lambda^2}\left\{(\lambda t_1+1)e^{-\lambda t_1}-(\lambda\mu+1)e^{-\lambda\mu}\right\}+e^{-\lambda\mu}\left\{(t_1-1)e^{t_1-\mu}-(\mu-1)\right\}\right]$$

$$+q\alpha\left\{(t_1-1)e^{t_1-\mu}-(\mu-1)\right\}-\frac{\alpha\beta}{\lambda}\left\{\frac{1}{\lambda^2}\left((\lambda t_2+1)e^{-\lambda t_2}-1\right)+\frac{t_2^2}{2}e^{-\lambda t_2}\right\} \quad \dots (16)$$

During period (0, T) total number of units holding (I_H) can be obtained as

$$I_H=\int_0^{\mu}I(t)dt+\int_{\mu}^{t_1}I(t)dt+\int_0^{t_2}I(t)dt$$

$$=\frac{(1-b)\beta}{\lambda+1}\left[\frac{1}{\lambda}(1-e^{-\lambda t_1})-(e^{\mu}-1)+\frac{\alpha e^{-\lambda\mu}}{\lambda(\lambda+1)}\left(\frac{1}{\lambda+1}+\mu+\frac{1}{\lambda}\right)-\frac{\alpha e^{-\lambda t_1}}{\lambda(\lambda+1)}\right.$$

$$\left.\left(\frac{1}{\lambda+1}+t_1+\frac{1}{\lambda}\right)-\left\{1+\alpha\left(\frac{\mu^2}{2}+\frac{\mu}{\lambda+1}+\frac{1}{(\lambda+1)^2}\right)\right\}e^{-\lambda\mu}(e^{t_1-\mu}-1)\right]$$

$$-\frac{\alpha}{2}\left\{e^{t_1-\mu}(t_1^2-2t_1+2)-(\mu^2-2\mu+2)\right\}\left\{\frac{(1-b)\beta}{\lambda+1}e^{-\lambda\mu}+q\right\}+q\left(1+\frac{\alpha\mu^2}{2}\right)$$

$$(e^{t_1-\mu}-1)-\frac{\beta}{\lambda}\left\{e^{-\lambda t_2}\left(\frac{1}{\lambda}(1+\alpha t_2^2)+\frac{2\alpha}{\lambda^2}\left(t_2+\frac{1}{\lambda}\right)+t_2+\frac{\alpha t_2^3}{3}\right)-\frac{1}{\lambda}\left(1+\frac{2\alpha}{\lambda^2}\right)\right\} \quad \text{... (17)}$$

Total amount of shortage units (I_S) during the period (0, T) is given by

$$I_S=-\int_0^{t_3} I(t)dt-\int_0^{t_4} I(t)dt$$

$$=\frac{\beta}{\lambda^2}\left[\left\{1+\gamma\left(\frac{2}{\lambda}-T\right)\right\}(e^{-\lambda t_3}+\lambda t_3-1)+\gamma t_3(e^{-\lambda t_3}-1)\right]$$

$$-\frac{(b-1)\beta}{\lambda(\lambda+1)}\left[\left\{1+\lambda\left(1-e^{-t_4}\right)\right\}e^{-\lambda t_4}-\lambda\right] \quad \text{... (18)}$$

And the total amount of lost sales (I_L) during the period (0, T) can be obtained as

$$I_L=\int_0^{t_3}\left\{1-\frac{1}{1+\gamma(t-T)}\right\}\beta e^{-\lambda t}dt=\frac{\beta\gamma}{\lambda^2}\left[e^{-\lambda t_3}\{\lambda(T-t_3)-1\}+1-\lambda T\right] \quad \text{... (19)}$$

Hence, the total average cost (K) of the inventory system is given by

$$K=\frac{1}{T}\left[C'+C_1 I_H+C_2 I_D+C_3 I_S+C_4.I_L\right] \quad \text{... (20)}$$

Equation (20) contains four variables t_1, t_2, t_3, t_4. However these variables are not independent and are related by (15). Also we have $K > 0$.
For minimum K, we must have

$$\frac{\partial K}{\partial t_1}=0 \text{ and } \frac{\partial K}{\partial t_4}=0, \quad \text{... (21)}$$

Equations (21) are equivalent to

$$\{1+f_1'(t_1)\}R-C_1T\left[\frac{(1-b)\beta}{\lambda+1}\left\{e^{-\lambda t_1}\left(1+\frac{\alpha}{(\lambda+1)^2}+\frac{\alpha t_1}{\lambda+1}\right)-\left(1+\alpha\left(\frac{\mu^2}{2}+\frac{\mu}{\lambda+1}+\frac{1}{(\lambda+1)^2}\right)\right)\right.\right.$$

$$\left.e^{t_1-(\lambda+1)\mu}\right\}-\frac{\alpha}{2}t_1^2e^{t_1-\mu}\left\{\frac{(1-b)\beta e^{-\lambda\mu}}{\lambda+1}+q\right\}+q\left(1+\frac{\alpha\mu^2}{2}\right)e^{t_1-\mu}+\beta t_2e^{-\lambda t_2}\left(1+\frac{\alpha t_2^2}{3}\right)$$

$$\left.f_1'(t_1)\right]-C_2T\left[\frac{(b-1)\alpha\beta t_1}{\lambda+1}\left\{e^{t_1-(\lambda+1)\mu}-e^{-\lambda t_1}\right\}+q\alpha t_1e^{t_1-\mu}+\frac{\alpha\beta t_2^2}{2}e^{-\lambda t_2}f_1'(t_1)\right]=0 \quad \text{... (22)}$$

and
$$\{1+f_2'(t_4)\}R-\frac{C_3T\beta}{\lambda^2}\left[\left\{1+\gamma\left(\frac{2}{\lambda}-T\right)\right\}\lambda\left(1-e^{-\lambda t_3}\right)f_2'(t_4)-\gamma\left(e^{-\lambda t_3}+\lambda t_3-1\right)\right.$$

$$\left.(1+f_2'(t_4))+\{\gamma e^{-\lambda t_3}(1-\lambda t_3)-\gamma\}f_2'(t_4)\right]-C_3\beta(b-1)T(e^{-t_4}-1)e^{-\lambda t_4}$$

$$-\frac{C_4\beta\gamma}{\lambda}\left[e^{-\lambda t_3}\{\lambda(t_3-T)+2\}f_2'(t_4)-\left(1+f_2'(t_4)\right)\right]=0 \quad \text{... (23)}$$

Where R is the total cost of the system (i.e. $R = KT$) and $f_1(t_1)$, $f_2(t_4)$ are given by equations (15). Solution of non-linear equations (22) and (23) is obtained with the help of MATHEMATICA 5.2.

4. NUMERICAL ILLUSTRATION:

To illustrate the model numerically the following parameter values are considered.

$b = 2$, $C' =$ Rs. 200 per order

$\beta = 50$, $C_1 =$ Rs. 3.0 per unit per month

$\lambda = 0.2$ unit, $C_2 =$ Rs. 10.0 per unit

$\alpha = 0.005$ unit, $C_3 =$ Rs. 12.0 per unit per month

$\mu = 1$ month, $C_4 =$ Rs. 5.0 per unit

$\gamma = 0.05$ unit $t_2 = 2$ months

$t_4 = 1$ month

then for the minimization of total average cost and with help of software MATHEMATICA 5.2 the optimal policy can be obtained such as $t_1 = 2.5067$ months, $t_3 = 0.9081$ month, $T = 6.4148$ months, $q = 79.1480$ units and $K =$ Rs. 211.5387 per month.

Effects of Backlogging Parameter (γ):

The backlogging parameter (γ) has been taken as 0.05. Now, after varying backlogging parameter from 0.04 to 0.06, the effects over the solution is observed.

γ	t_1	t_3	T	q	K
0.0400	2.5067	0.8285	6.3352	79.1480	214.6179
0.0425	2.5067	0.8469	6.3536	79.1480	213.8957
0.0450	2.5067	0.8663	6.3730	79.1480	213.1440
0.0475	2.5067	0.8866	6.3933	79.1480	212.3584
0.0500	2.5067	0.9081	6.4148	79.1480	211.5387
0.0525	2.5067	0.9307	6.4374	79.1480	210.6801
0.0550	2.5067	0.9547	6.4614	79.1480	209.7810
0.0575	2.5067	0.9801	6.4868	79.1480	208.8367
0.0600	2.5067	1.0072	6.5139	79.1480	207.8441

The study of above table reveals the some interesting facts. With the increment in backlogging parameter there is increment in time parameters t_3 and T while there is no effect on the parameters t_1 and q. Total average cost of the system decreases progressively with the increment in backlogging parameter.

Effects of Deterioration Parameter (α):

Initially, the deterioration parameter (α) has been taken as 0.005. We observe the following effects with the variation in deterioration parameter from 0.001 to 0.009.

α	t_1	t_3	T	q	K
0.001	2.3945	0.8999	6.2944	79.1480	186.2618
0.002	2.4233	0.9020	6.3253	79.1480	192.3892
0.003	2.4516	0.9040	6.3556	79.1480	198.6450
0.004	2.4794	0.9061	6.3855	79.1480	205.0297
0.005	2.5067	0.9081	6.4148	79.1480	211.5387
0.006	2.5335	0.9101	6.4436	79.1480	218.1707
0.007	2.5599	0.9120	6.4719	79.1480	224.9435
0.008	2.5859	0.9140	6.4999	79.1480	231.8575
0.009	2.6114	0.9159	6.5273	79.1480	238.8876

From the above table we observe some interesting facts, which are quite obvious when considered in the light of reality. We notice that with the increment in deterioration parameter there is increment in time parameters t_1, t_3, T and total average cost of the system while there is no effect on the parameters q.

Effects of Demand Parameter (λ):

The demand parameter (λ) has initially been taken as 0.2. Now, we attempt to study the effects of change in this parameter from 0.12 to 0.28 and observe the effects it has over the solution.

λ	t_1	t_3	T	q	K
0.12	3.1920	0.9559	7.1479	81.7572	402.7423
0.14	2.9676	0.9397	6.9073	81.0931	324.5968
0.16	2.7865	0.9270	6.7135	80.4370	273.4882
0.18	2.6356	0.9167	6.5523	79.7886	237.7688
0.20	2.5067	0.9081	6.4148	79.1480	211.5387
0.22	2.3943	0.9006	6.2949	78.5149	191.5097
0.24	2.2949	0.8942	6.1891	77.8893	175.7831
0.26	2.2059	0.8885	6.0944	77.2710	163.1377
0.28	2.1253	0.8833	6.0086	76.6601	152.7582

The study of above table reveals the some interesting facts. We notice that parameters t_1, t_3, T, q and total average cost of the system decreases progressively with the increment in demand parameter (λ).

Effects of Life Time Parameter (μ):

Initially, the life time parameter (μ) has been taken as 1.0. We observe the following effects with the variation in this parameter from 0.6 to 1.4.

μ	t_1	t_3	T	q	K
0.6	2.5077	0.9082	6.4159	38.9666	212.4006
0.7	2.5075	0.9081	6.4156	47.6831	212.1655
0.8	2.5073	0.9081	6.4154	57.2249	211.9538
0.9	2.5070	0.9081	6.4151	67.6805	211.7397
1.0	2.5067	0.9081	6.4148	79.1480	211.5387
1.1	2.5063	0.9080	6.4143	91.7353	211.3246
1.2	2.5058	0.9080	6.4138	105.5620	211.0941
1.3	2.5053	0.9080	6.4133	120.7602	210.8604
1.4	2.5048	0.9079	6.4127	137.4757	210.6164

From the above table we observe some interesting facts, which are quite obvious when considered in the light of reality. We notice that parameters t_1, t_3, T and total average cost of the system decreases while the parameter q increases with the increment in life time parameter.

5. CONCLUSION:

In this paper a production inventory model is formulated and analyzed for variable rate of deterioration with the concept of life period of items. The demand for the product is taken as exponential decreasing function of time and the production rate is the linear combination of on-hand inventory and demand rate at any instant. The backlogging rate is taken as variable and inversely proportional to the waiting time up to the arrival of next lot. Cost minimization technique is used to get the expressions for total cost and other parameters. We could extend the deterministic demand function to stochastic fluctuating demand patterns in the proposed model and we could generalized the model to allow for quantity discounts, inflation and others.

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